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to us for the first time the striking dissimilarity in chemical behavior between these two types of isomeric compounds.

It seems safe to conclude, therefore, from the results of this new work that any isothiocyanate formed by interaction of potassium thiocyanate with a secondary chloracetanilide will be found to be unstable and easily transformed into its corresponding thiohydantoin. Whether Beckurts and Frerichs were actually dealing in their work with such normal thiohydantoins instead of the pseudo modifications must be decided by further investigation. In the case of tertiary anilides of isothiocyanacetic $\text{SCN}.\text{CH}_2\text{CO.NR}_2$, such cyclic rearrangements cannot take place, and in such cases we expect to obtain acyclic combinations containing the grouping NCS functioning as a true isothiocyanate. Investigations dealing with various phases of this interesting problem are now in progress and the results will be published in the *Journal of the American Chemical Society*.

¹ Wheeler and Merriam, *J. Amer. Chem. Soc.*, **23**, 1901 (283).

² *Amer. Chem. J.*, **28**, 1902 (121); Johnson, *J. Amer. Chem. Soc.*, **25**, 1903 (483).

³ *Arch. Pharm.*, **253** (233); *Chem. Zentrbl.*, **2**, 1915 (614); *Chem. Abstracts*, **10**, 1916, (888).

⁴ Aschan, *Ber. chem. Ges.*, **17**, 1884 (420); Marckwald, Neumark and Stelzner, *Ibid.*, **24**, 1891 (3278).

⁵ *Loc. cit.*

STUDIES OF MAGNITUDES IN STAR CLUSTERS. XI. FREQUENCY OF CURVES THE ABSOLUTE MAGNITUDE AND COLOR INDEX FOR 1152 GIANT STARS.

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Communicated by G. E. Hale, April 8, 1920

Kapteyn, Schwarzschild, Parkhurst, Russell, and other investigators of statistical problems in stellar astronomy have paid special attention to the relative frequency of successive values of the absolute brightness of stars (the luminosity law) and the relative frequency of different spectral or color types. The laws of the frequencies of absolute magnitude and spectrum are indeed fundamental in studies of stellar evolution and the arrangement of stars in space; but to determine these laws, at the same time keeping the errors due to unavoidable and vitiating selection of data at a minimum, is by no means a simple process. Insufficient knowledge of stellar distances, and frequently of apparent magnitudes and spectra as well, presents a serious obstacle. In particular, it is difficult to obtain satisfactory luminosity curves for each spectral type, or representative spectral curves for small and clearly defined intervals of absolute magnitude. The luminosity curve that does not differentiate

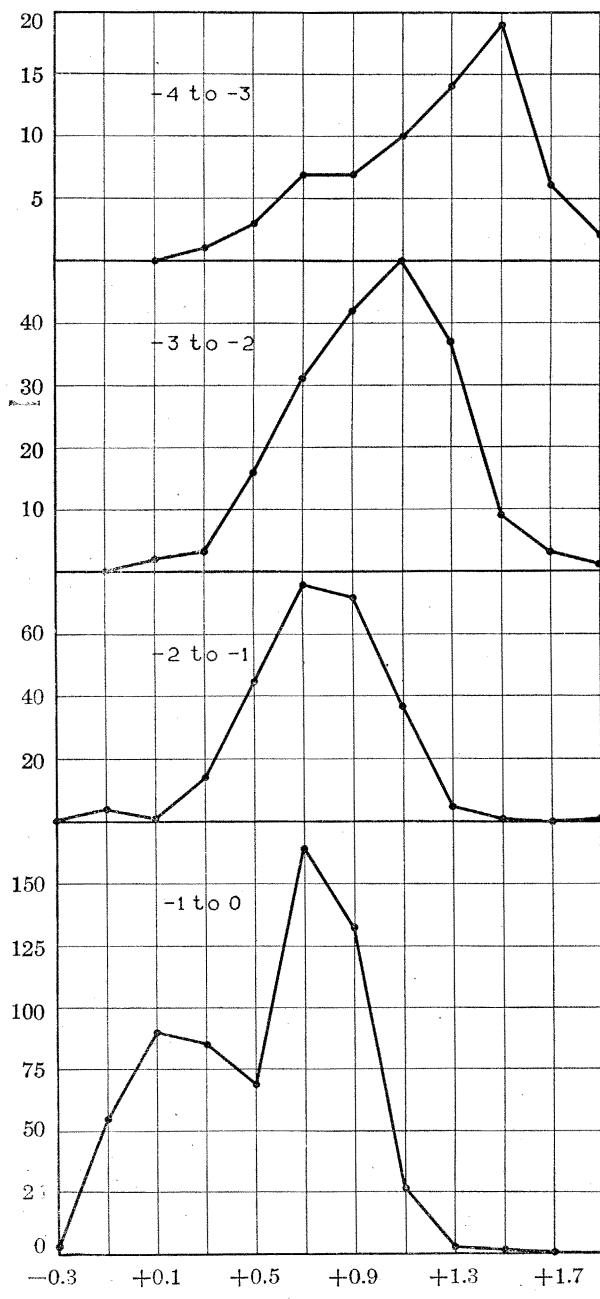


FIG. 1

Curves of the frequency of color index for successive intervals of absolute magnitude. Ordinates are numbers of stars; abscissae are color indices.

spectral types conceals the important differences for red and blue stars of the giant-dwarf phenomena; and the spectral (or color) curve that includes a great range of absolute magnitude is of limited value because

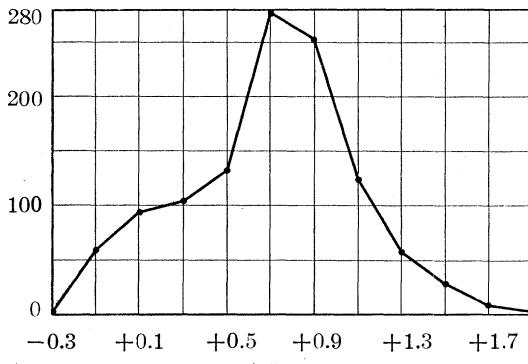


FIG. 2

Integrated color curve for 1145 giant stars of all absolute magnitudes brighter than zero. Coördinates as in Fig. 1.

it integrates indiscriminately the various masses, densities, ages, and other factors that appear to change with luminosity.

The study of magnitudes and colors in star clusters affords, however, for the brightest of giant stars, a fairly accurate determination of the frequencies of magnitudes and colors for small intervals of color and

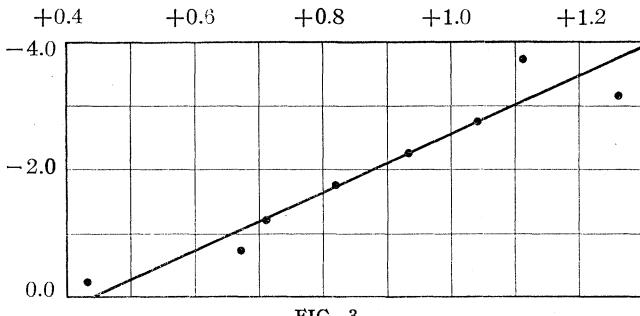


FIG. 3

Change of mean color index with brightness for 1143 giant stars in clusters. Ordinates are absolute photovisual magnitudes; abscissae are color indices. (See last three columns of Table I.)

magnitude, respectively. The relative apparent magnitudes and distances, which in most cases are prerequisite to a knowledge of absolute brightness, are sufficiently known in nine systems to permit a combination of all observations for a statistical investigation of the above-mentioned astrophysical laws.

The discussion of the observational data for 1152 stars brighter than absolute photovisual magnitude zero is summarized in the tabular results and illustrative curves of the present note.

The clusters whose stars are included in this synopsis of absolute magnitude are Messier 3, 5, 11, 13, 15, 30, 68; N. G. C. 4147, 7006. Their distances range between seven and seventy thousand parsecs; the apparent magnitudes that correspond to zero absolute magnitude range between 14.3 and 19.2.

The available absolute magnitudes fainter than zero are less accurate than those for the brighter giants, and for many of the most distant

TABLE I
DATA FOR CURVES OF COLOR FREQUENCY
(Tabulated quantities are numbers of stars)

ABSOLUTE PHOTOVISUAL MAGNITUDE	COLOR INDEX										ALL COLORS	MEAN MAGNI- TUDE	MEAN COLOR INDEX	
	-0.2 to -0.2	-0.2 to 0.0	0.0 to 0.2	0.2 to 0.4	0.4 to 0.6	0.6 to 0.8	0.8 to 1.0	1.0 to 1.2	1.2 to 1.4	1.4 to 1.6	1.6 to 1.8	1.8 to 2.0		
	-0.4 to -0.2	-0.2 to 0.0	0.0 to 0.2	0.2 to 0.4	0.4 to 0.6	0.6 to 0.8	0.8 to 1.0	1.0 to 1.2	1.2 to 1.4	1.4 to 1.6	1.6 to 1.8	1.8 to 2.0		
-5.0 to -4.5	0	0	0	0	0	0	0	0	0	0	0	0	0	
-4.5 to -4.0	0	0	0	1	0	0	0	1	0	0	0	0	2	-4.38 +0.77
-4.0 to -3.5	0	0	0	1	2	2	2	3	3	6	1	0	20	-3.74 +1.11
-3.5 to -3.0	0	0	0	0	1	5	5	7	11	13	5	2	49	-3.18 +1.26
-3.0 to -2.5	0	0	1	1	7	14	13	21	25	6	1	1	90	-2.77 +1.04
-2.5 to -2.0	0	0	1	2	9	17	29	29	12	1	2	0	102	-2.25 +0.93
-2.0 to -1.5	0	0	1	4	22	16	39	24	3	1	0	0	110	-1.76 +0.82
-1.5 to -1.0	0	4	0	10	23	60	33	13	2	0	0	1	146	-1.22 +0.71
-1.0 to -0.5	1	1	16	34	18	60	77	18	1	1	0	0	227	-0.72 +0.67
-0.5 to 0.0	2	54	74	51	50	104	55	8	1	0	0	0	399	-0.23 +0.43
All magnitudes	3	59	93	104	132	278	253	124	58	28	9	4	1145	-1.18 +0.69

clusters the data for the fainter stars are very incomplete. The present study therefore is limited to stars brighter than zero magnitude, and for the luminosity curves it is restricted to the clusters Messier 3, 11, and 13, for which we have published catalogues of magnitudes and colors complete to the adopted fainter limit.

For intervals of 0.2 mag. in color index and of 0.5 mag. in absolute photovisual magnitude table I contains the number of stars from all nine clusters. Only seven of the 1152 stars fall outside the limits of this table. Their absolute magnitude and color indices are:

-5.45	+1.01	-2.77	+2.06
-4.14	+1.23	-2.19	-0.52
-5.09	+1.00	-1.03	-0.60
-5.01	+1.83		

It should be especially noted that only six stars (four of those above and two in table I) are brighter than -4.0 , and the brightest recorded star is fainter than -5.5 . We are able, moreover, to go much farther with this inference relative to an upper limit for stellar luminosity. The investigations carried on at Mount Wilson during the last six years have yielded provisional but sufficiently reliable absolute magnitudes in more than fifty clusters; they permit the statement that, *of more than a million stars, less than a tenth of 1% are brighter than -4 , photovisually*. Since the brightest giants are red, the corresponding upper limit for absolute photo-

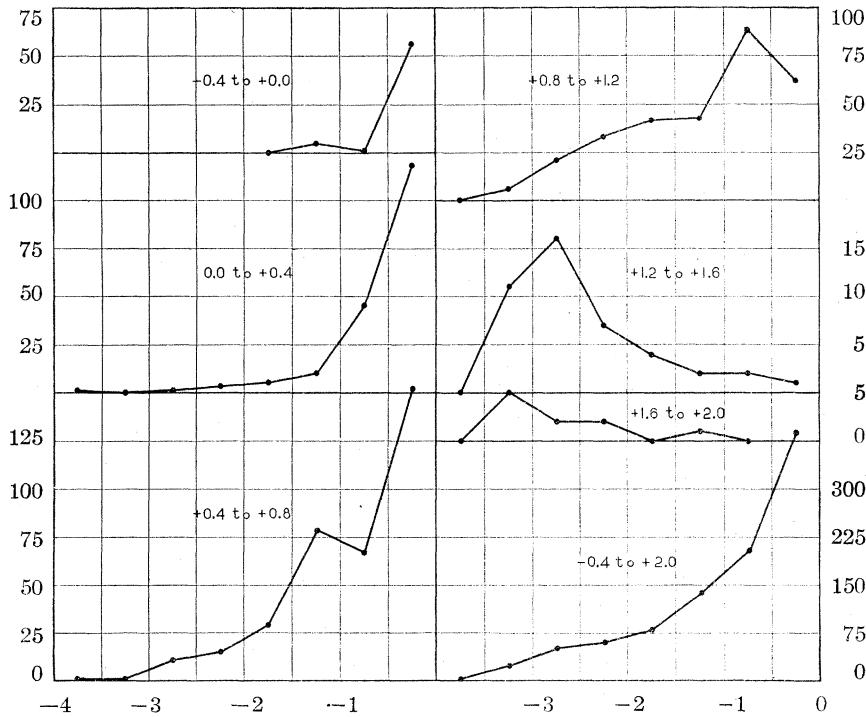


FIG. 4

Luminosity curves for different intervals of color index (Table II). Ordinates are numbers of stars; abscissae are absolute photovisual magnitudes.

graphic magnitude may be placed at -3 . There appears to be scarcely a single star brighter than photovisual magnitude -6 .

The existence of a maximum luminosity, indicated by these results, is probably closely related to the upper limits of stellar mass suggested by Eddington's researches on giant stars; it is highly significant in considerations of external galactic systems.

It is also of interest that of the 1152 stars only three may be considered slightly abnormal in color, in that they fall outside the limits of table I. This speaks for the uniformity of stars throughout the universe, and

for the comparability of the spectral phenomena in globular clusters with those of the solar neighborhood.

Color frequency-curves for intervals of one magnitude are shown in figures 1 and 2, based on the data of table I. The striking progression of maximum frequency toward the blue, with decreasing magnitude, again emphasizes the law of average color for giant stars in clusters, viz., *the brighter the giant star the greater its color index*. The progression of average color with luminosity is also clearly shown by the means in the last two columns of table I, which are plotted in figure 3, the six stars brighter than -4.0 being omitted.

A double maximum appears in the color curve for stars between magnitudes 0.0 and -1.0 . It is not the result of the combination of observa-

TABLE II
DATA FOR LUMINOSITY CURVES
(Tabulated quantities are numbers of stars)

ABSOLUTE PHOTOVISUAL MAGNITUDE	COLOR INDEX										ALL COLORS		
	-0.4 to -0.2	-0.2 to 0.0	0.0 to +0.2	+0.2 to +0.4	+0.4 to +0.6	+0.6 to +0.8	+0.8 to +1.0	+1.0 to +1.2	+1.2 to +1.4	+1.4 to +1.6	+1.6 to +1.8	+1.8 to +2.0	
-5.0 to -4.5	0	0	0	0	0	0	0	0	0	0	0	0	0
-4.5 to -4.0	0	0	0	0	0	0	0	0	0	0	0	0	0
-4.0 to -3.5	0	0	0	1	0	1	0	0	0	0	0	0	2
-3.5 to -3.0	0	0	0	0	0	1	3	3	6	5	3	2	23
-3.0 to -2.5	0	0	0	1	1	10	8	13	11	5	1	1	51
-2.5 to -2.0	0	0	1	2	7	8	17	16	6	1	2	0	60
-2.0 to -1.5	0	0	1	4	20	9	25	17	3	1	0	0	80
-1.5 to -1.0	0	4	0	10	22	56	32	11	2	0	0	1	138
-1.0 to -0.5	0	1	15	30	13	54	71	17	1	1	0	0	203
-0.5 to 0.0	2	54	71	47	49	103	54	8	1	0	0	0	389
All magnitudes	2	59	88	95	112	242	210	85	30	13	6	4	946

tions from clusters at different stages of development, because it is shown separately for Messier 3 and 13. It may be due to unequal evolutionary values for the different intervals of color—to a relatively speedier development, that is, throughout the interval of color index $+0.2$ to $+0.6$, for stars of a certain mass.

Table II, similar in form to table I, contains data for the luminosity curves (fig. 4), showing the distribution in color and absolute magnitude of 946 stars from Messier 3, 11, and 13. These curves, for intervals of 0.4 mag. in color index, indicate that our material gives the complete distribution of luminosity for the red giants, but that for the blue stars the maximum frequency falls below zero absolute magnitude.

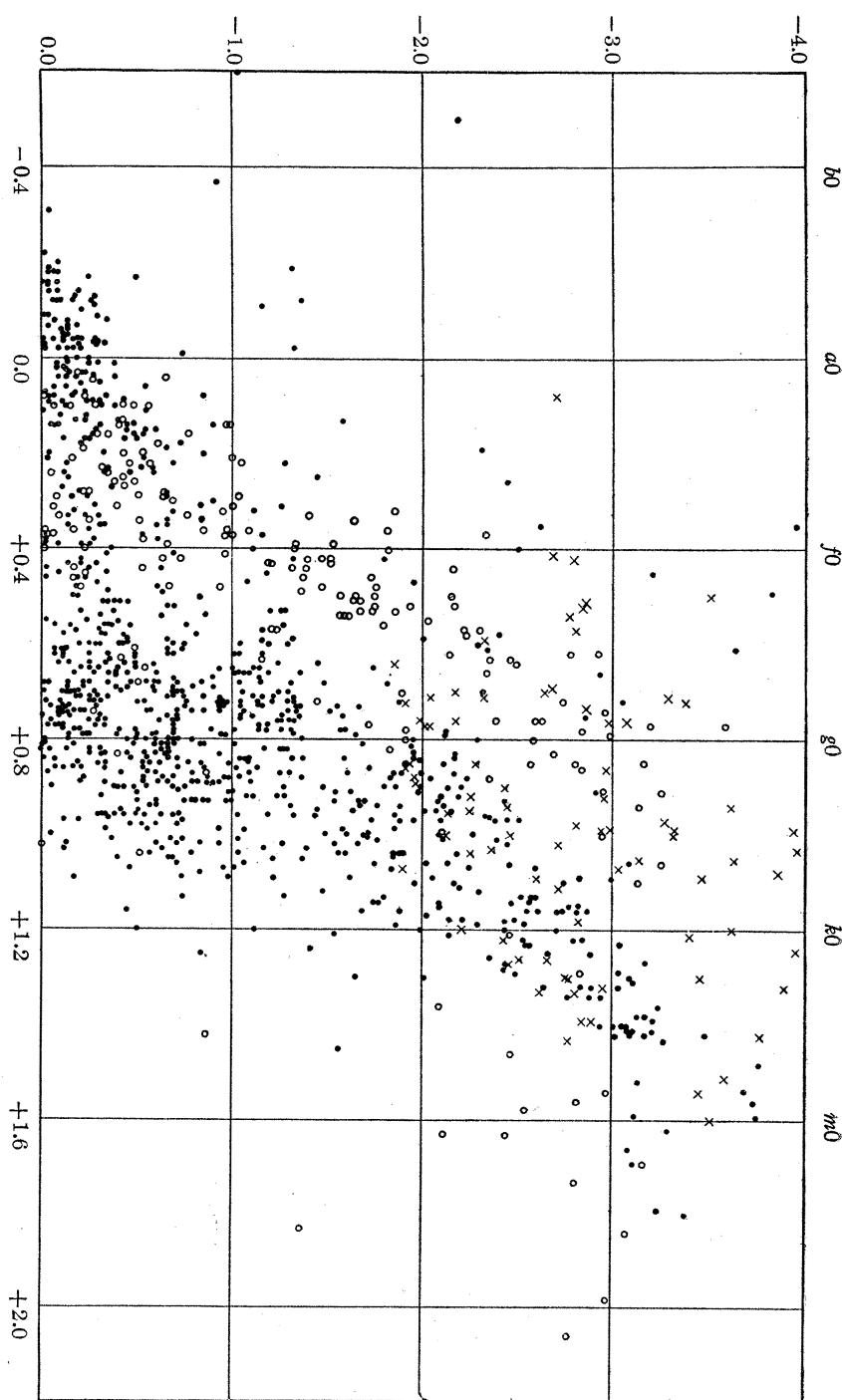


FIG. 5

Distribution of giant stars in color and brightness. Ordinates are absolute photovisual magnitudes; abscissae are color indices with the corresponding color class indicated at the top of the diagram. (See last paragraph of text for further explanation.)

It is believed that the average probable error of the paralaxes of these nine clusters is much less than 15%. Suppose, however, that for one of the clusters the true distance differs by 30% from the adopted value. The adopted absolute magnitudes for that cluster would be systematically in error by about 0.6 mag. The position of the maxima of the luminosity curves for the nine clusters together would be slightly altered, therefore, if the supposed error were uncompensated; but the general form of none of the curves would be materially changed.

Figure 5 represents the distribution in color and luminosity of the individual stars, the circles representing data from Messier 11, the dots giving results for the most accurately measured clusters (Messier 3, 5, 13, 15, and 68), and the crosses representing the stars for the clusters for which the magnitudes depend upon less extensive investigations. Together with the luminosity and color curves, this diagram illustrates the present state of our information concerning the giant stars in clusters.

*ON THE DISTORTION IN CONFORMAL MAPPING WHEN THE
SECOND COEFFICIENT IN THE MAPPING FUNC-
TION HAS AN ASSIGNED VALUE*

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Communicated by E. H. Moore, April 27, 1920

Note III On Conformal Mapping Under Aid of Grant No. 207 From the Bache Fund

Let $w = z + a_2z^2 + \dots + a_nz^n + \dots$ be a power series in z convergent for $|z| < 1$ and such that the circle $|z| < 1$ is mapped conformally on a simple (that is, simply connected and nowhere overlapping) region in the w -plane. Koebe¹ has shown that on the circumference $|z| = r$, where $0 < r < 1$, the distortion $|dw/dz|$ and also $|z|$ lie between positive bounds depending on r alone, and the writer² has determined the exact values of these bounds. A far more difficult problem arises when some of the coefficients of the power series are given *a priori*. The simplest case where $a_2 = ae^{\gamma i}$ ($a \geq 0$) is given was investigated by the writer,³ the method employed failing, however, to furnish the upper bound of $|z|$ in the case $0 \leq a < 1$. This defect has now been remedied, and denoting by $r(a)$, for $0 \leq a < 1$, the root between zero and unity of the equation

$$\frac{2r}{1 + 2(a-1)r + r^2} - \log \frac{1+r}{1-r} = 0,$$

and by $\cos \beta$, for $0 < r \leq r(a)$, the positive root of the equation

$$\frac{2r}{1 - 2r\cos \beta + r^2} - \log \frac{1+r}{1-r} = 0,$$

we have the following: